PDE-based Equivariant Neural Networks

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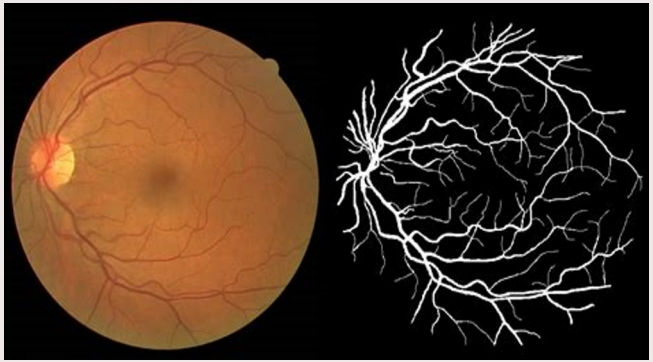
Starting point: Convolutional Neural Networks

Applications of interest:

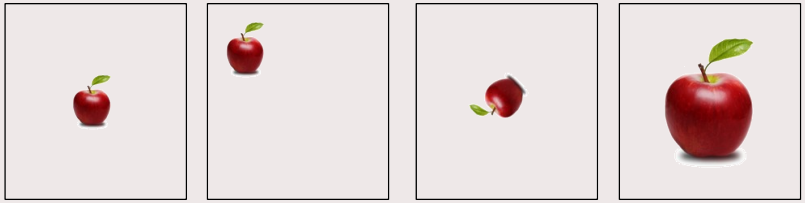
- Image segmentation
- Image classification



Retinal vessel segmentation

Digit classification

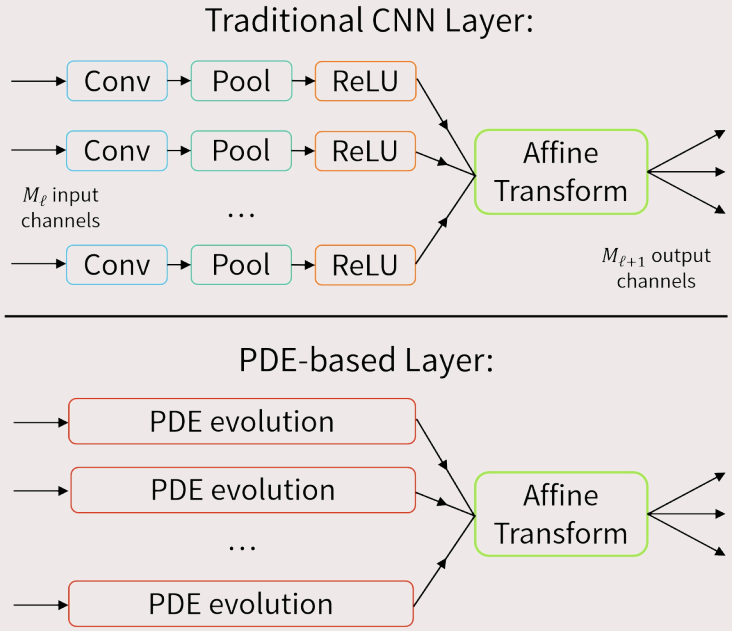
Equivariance



Group-CNN’s (G-CNN)

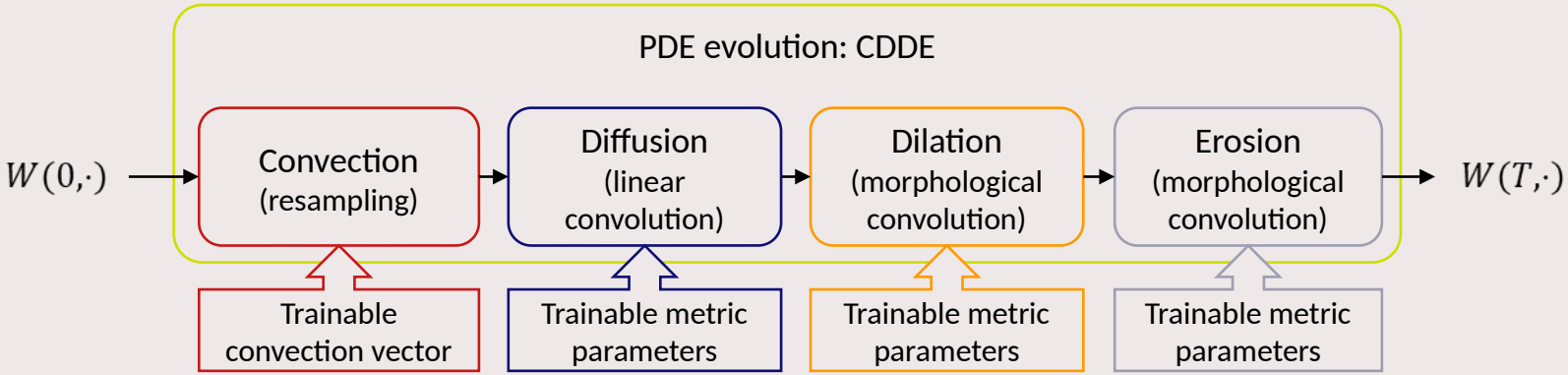
	Translation	Rotation	Scaling
Spatial CNN	✓		
SE(d) CNN	✓	✓	
SIM(d) CNN	✓	✓	✓

Our work: PDE-based networks



PDE of interest

$$\frac{\partial W}{\partial t} = \quad -\boldsymbol{c} \cdot \nabla W \qquad -\left(-\nabla_{\mathcal{G}_1} W\right)^{\alpha} \quad + \left\|\nabla_{\mathcal{G}_2} W\right\|_{\mathcal{G}_2}^{2\alpha} \quad - \left\|\nabla_{\mathcal{G}_3} W\right\|_{\mathcal{G}_3}^{2\alpha}$$



Parameter interpretation

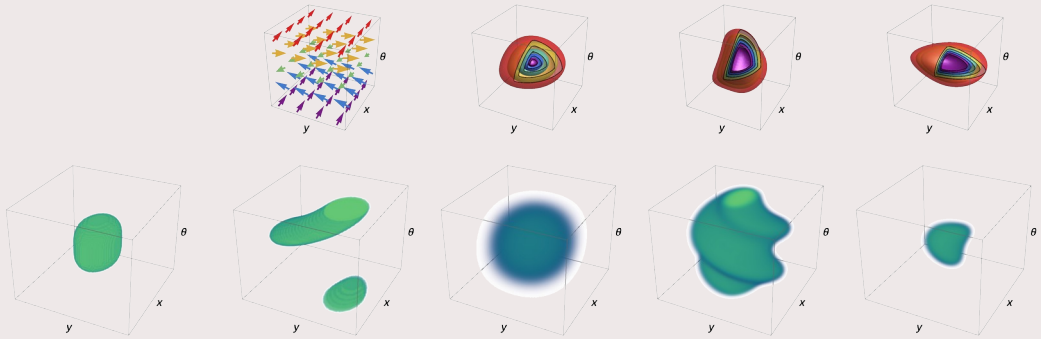
translation equivariance

$\mathbb{R}^2$	Transport	Regularization	Max pooling	Min pooling
PDE Term:	$-\boldsymbol{c}W$	$-(-\Delta_{\mathcal{G}_1})^{\alpha}W$	$+\left\|\nabla_{\mathcal{G}_2^+}W\right\|_{\mathcal{G}_2^+}^{2\alpha}$	$-\left\|\nabla_{\mathcal{G}_2^-}W\right\|_{\mathcal{G}_2^-}^{2\alpha}$
Parameters:	transport vector $\begin{pmatrix}100\\120\end{pmatrix}$	Riemannian metric tensor $\begin{pmatrix}1&0\\0&1\end{pmatrix}$	Riemannian metric tensor $\begin{pmatrix}9&0\\0&1\end{pmatrix}$	Riemannian metric tensor $\begin{pmatrix}5&11\\11&40\end{pmatrix}$
Operation:	resample with offset	convolution with kernel	dilation with kernel	erosion with kernel

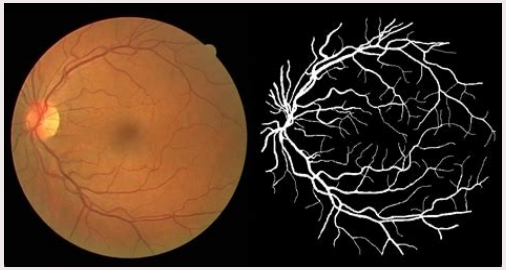
Parameter interpretation

translation & rotation equivariance

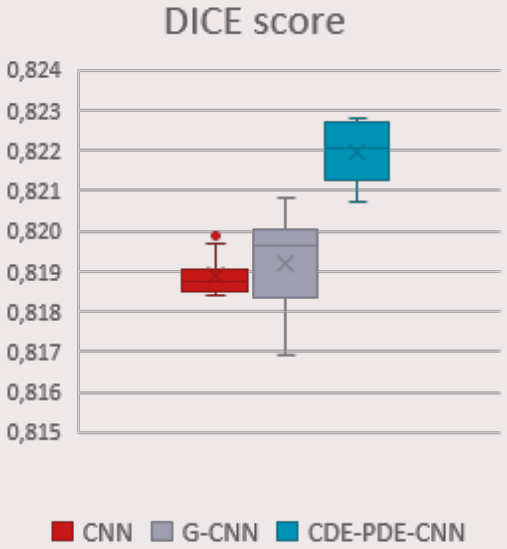
$\mathbb{M}_2$	Transport	Regularization	Max pooling	Min pooling
PDE Term:	$-\boldsymbol{c}W$	$-\left(-\Delta_{\mathcal{G}_1}\right)^{\alpha}W$	$+\left\|\nabla_{\mathcal{G}_2^+}W\right\|^{2\alpha}_{\mathcal{G}_2^+}$	$-\left\|\nabla_{\mathcal{G}_2^-}W\right\|^{2\alpha}_{\mathcal{G}_2^-}$
Parameters:	G–invariant vector field $\begin{pmatrix}1\\0\\2\end{pmatrix}$	Riemannian metric tensor $\begin{pmatrix}4&0&0\\0&6&0\\0&0&1\end{pmatrix}$	Riemannian metric tensor $\begin{pmatrix}4&0&0\\0&9&0\\0&0&0.5\end{pmatrix}$	Riemannian metric tensor $\begin{pmatrix}6&0&0\\0&3&0\\0&0&1.5\end{pmatrix}$
Operation:	resample with offset	convolution with kernel	dilation with kernel	erosion with kernel



Experiment: Retinal Vessel Segmentation (DRIVE)

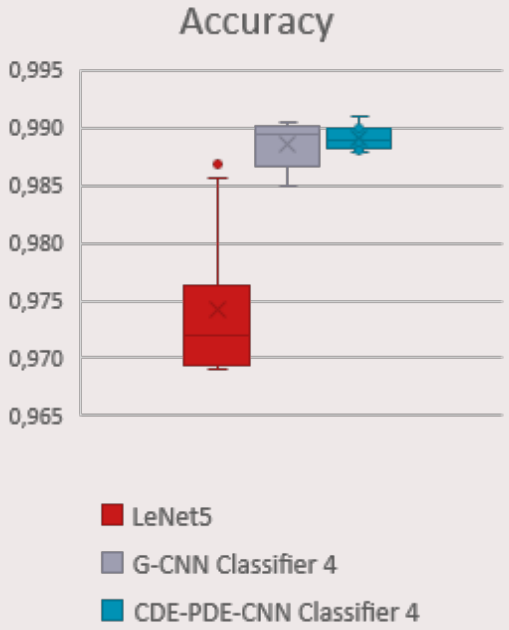


Network (12 layers)	CNN	G-CNN	CDE-PDE-CNN
parameters	129.432	114.378	3.678
test time (s)	2,2	14,1*	9,8*
equivariance	translation	translation + rotation	translation + rotation
non-linearity	ReLU	ReLU	trainable PDE-based



Experiment: Digit Classification (RotNIST)

Network	LeNet5	G-CNN Classifier 4	CDE-PDE-CNN Classifier 4
parameters	44.426	12.700	2.542
test time (s)	0,10	0,86*	0,70*
equivariance	translation	translation + rotation	translation + rotation
non-linearity	ReLU	ReLU	trainable PDE-based



Details

Smets, Portegies, Bekkers, Duits.
PDE-based Group Equivariant Convolutional Neural Networks.
arXiv preprint https://arxiv.org/abs/2001.09046, to appear in Journal of Mathematical Imaging and Vision (SSVM 2021 Special Issue).

Future

- Studying properties of PDE-G-CNN's
- Several promising directions in evolving PDE-based CNN's

- Interested in contact with people that see promising applications for these types of networks in their domains